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High-precision studies of domain-wall properties in the 2D Gaussian Ising spin glass

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Abstract.
In two dimensions, short-range spin glasses order only at zero temperature, where efficient combinatorial optimization techniques can be used to study these systems with high precision. The use of large system sizes and high statistics in disorder averages allows for reliable finite-size extrapolations to the thermodynamic limit. Here, we use a recently introduced mapping of the Ising spin-glass ground-state problem to a minimum-weight perfect matching problem on a *sparse* auxiliary graph to study square-lattice samples of up to $10\,000 \times 10\,000$ spins. We propose a windowing technique that allows to extend this method, that is formally restricted to planar graphs, to the case of systems with fully periodic boundary conditions. These methods enable highly accurate estimates of the spin-stiffness exponent and domain-wall fractal dimension of the 2D Edwards-Anderson spin-glass with Gaussian couplings. Studying the compatibility of domain walls in this system with traces of stochastic Loewner evolution (SLE), we find a strong dependence on boundary conditions and compatibility with SLE only for one out of several setups.

1. Introduction

Spin glasses are by now a mature subject with a multitude of spin-off directions including applications in neural networks and machine learning, error correcting codes and gene regulation [1, 2]. After more than half a century of research, some of the fundamental questions are still unsolved, including the nature of the ordering in the spin-glass phase. Although many more complicated models including systems with continuous spins [3–7], long-range interactions [8] or samples subject to external fields [9] have been studied, the most fundamental questions are mostly studied for the paradigmatic example of the short-range Ising spin glass.

A spin glass phase has not been observed in a two-dimensional system with short-range interactions, and in such systems spin glass is confined to zero temperature which is hence the critical temperature of this system. Nevertheless, the physics of the 2D Ising spin glass is rather rich and many features are still under active debate [10–15]. One of its most convenient features is that in contrast to systems in more than two dimensions ground states can be found in polynomial time using efficient algorithms [16–18]. Here we report on advances in the machinery of such combinatorial optimization algorithms that allow us to determine the critical exponents characterizing the behavior of the Gaussian Ising spin glass in two dimensions for large system sizes and thus arrive at estimates of these critical parameters that are of unprecedented accuracy.

2. Model and methods

We consider the standard Edwards-Anderson Ising spin glass model with Hamiltonian [19]

$$\mathcal{H} = -\sum_{\langle i,j \rangle} J_{ij} s_i s_j. \tag{1}$$

For the purposes of this paper, the spins are located at the vertices of a square lattice, where interactions are only between pairs of nearest neighbors. We consider a number of different boundary conditions: if periodic boundaries are chosen in one direction only while free boundaries are assumed in the other (PFBC), the graph remains planar, which is in contrast to the case of periodic-periodic boundaries (PPBC), which correspond to a torus topology. In both cases, the common periodic direction is switched to antiperiodic for the defect-energy calculations reported below. In the present work we focus on the Gaussian distribution of exchange couplings, i.e.,

$$P(J_{ij}) = \frac{1}{\sqrt{2\pi}} \exp\left[-J_{ij}^2/2\right].$$

While the ground-state problem for Ising systems in three dimensions and above has been shown to be *NP* hard [20], this is not the case in two dimensions, where ground states [16, 21] and even partition functions [22] can be computed in polynomial time. The ground-state algorithm of Ref. [16] is based on the concept of defect lines connecting frustrated plaquettes. As was initially realized by Toulouse [23], the elementary closed loops of a lattice carry the basic information about the frustration state of a spin configuration, and one calls a plaquette frustrated if it has an odd number of antiferromagnetic bonds, such that it is impossible to satisfy all the interactions with a spin configuration around the loop. As a consequence, the ground state of an Ising system corresponds to a complete pairing of frustrated plaquettes along paths on the dual lattice that traverse a minimal sum of $|J_{ij}|$, a problem known as minimum-weight perfect matching [24]. This is a polynomial problem solved by the blossom algorithm introduced by Edmonds [25] and further improved in a number of generations up to the most recent Blossom V version of the code that is used here [26].

While this mapping to a matching problem transforms the Ising spin-glass ground-state problem into a practically tractable task, there remain a number of problems: Firstly, the matching of plaquettes formally requires a calculation on the complete graph of frustrated plaquettes, thus turning a problem on an originally sparse graph into an auxiliary calculation on a dense graph with the resulting overheads in memory and time to solution. Secondly, as a consequence of this situation, it is a well-established practice to thin out this dense graph to only include matching edges up to a certain maximum weight as very distant frustrated plaquettes are very unlikely to be part of the matching of overall minimum weight [16, 27, 28]. As a result, strictly speaking this approach does no longer guarantee to yield exact ground states as there always remains a certain small, but non-zero probability for rare events to occur. Thirdly, the approach of Ref. [16] only applies to samples on planar graphs. On non-planar graphs the solution of the auxiliary matching problem is not guaranteed to correspond to a realizable spin configuration, such that a ground-state cannot always be found.

These problems are addressed by the approach used here as first discussed in the more comprehensive report of Ref. [29]. It relies on a different representation of the spin system as a combinatorial optimization problem that was first proposed in Refs. [17, 18]. In this model, the dual graph of the original spin lattice is expanded into an auxiliary graph where each plaquette of the spin lattice corresponds to a complete graph of four nodes also known as a Kasteleyn city. Once the Kasteleyn cities are contracted back to individual vertices, a perfect matching on this auxiliary graph is seen to correspond to a set of closed loops on the dual of the spin lattice separating up from down spins. Writing the task to find a spin-glass ground state as a
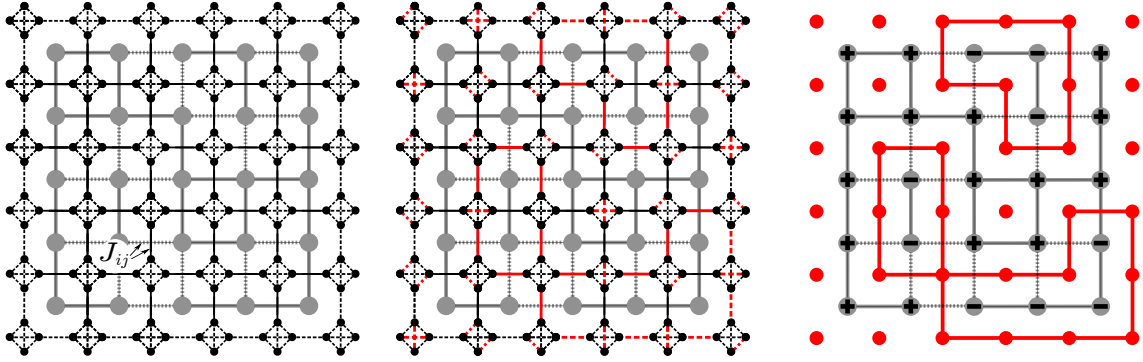
Figure 1. Illustration of the transformation linking the Ising spin-glass ground-state calculation to a minimum-weight perfect matching problem on an auxiliary graph. Left: Expansion of the Ising graph (light grey) to an auxiliary graph of Kasteleyn cities for each plaquette of the original lattice. The edge weights are chosen as J_{ij} for links corresponding to the original lattice bonds and 0 otherwise. Middle: A minimum-weight perfect matching on the auxiliary graph. Right: On contracting back the Kasteleyn cities to individual vertices, the matching corresponds to a set of closed contours on the dual of the original lattice. In the ground-state spin configuration these contours separate up from down spins.

minimum-cut problem, it is seen that in this setup a spin-glass ground state corresponds to a minimum-weight perfect matching on the auxiliary graph if the weights of physical edges are chosen to be identical to the couplings J_{ij} and the weights of auxiliary edges are assumed to be zero [18]. The whole transformation is illustrated in Fig. 1. The matching problem solved in this case remains to be on a sparse graph, hence allowing to treat much larger system sizes than with the model proposed by Bieche *et al.* [16]. In combination with an efficient implementation of the matching algorithm [26], the new mapping allowed us to treat systems up to $10\,000 \times 10\,000$ spins

While the new transformation leads to much more efficient ground-state calculations for planar graphs, the ambiguity and resulting non-applicability for non-planar lattices still remains. For a system with periodic boundary conditions, for instance, some of the loops identified by the matching (cf. the right panel of Fig. 1) will wrap around the lattice in one of the two inequivalent directions and, as a consequence, on traversing across the system in one of the periodic directions one will cross an odd number of such loop lines, and a spin configuration consistently changing sign on crossing such lines is then impossible to find. As a workaround, in Ref. [29] we proposed a windowing technique that allows to compute ground states with high reliability also for systems with periodic boundaries. It proceeds by computing an exact ground state using the minimum-weight perfect matching (MWPM) approach on a square sub-system or "window" of size $L' \times L'$ while keeping all spins outside of the window fixed. This can be achieved by choosing sufficiently strong bonds between the spins just outside of the window that keep them in the chosen relative orientation. The setup is illustrated in the left panel of Fig. 2. In practice, we find that a maximal window size, namely $L' = L - 2$ yields the best results. A single such computation does not always yield a ground-state, but repeated calculations on windows that are randomly shifted over the lattice yields a ground state with a probability that weakly depends on the number of iterations (with about 80% probability for 30 iterations) and that is practically independent of system size. The success probability can be arbitrarily increased by additional repetitions of the whole procedure as the total success probability of m
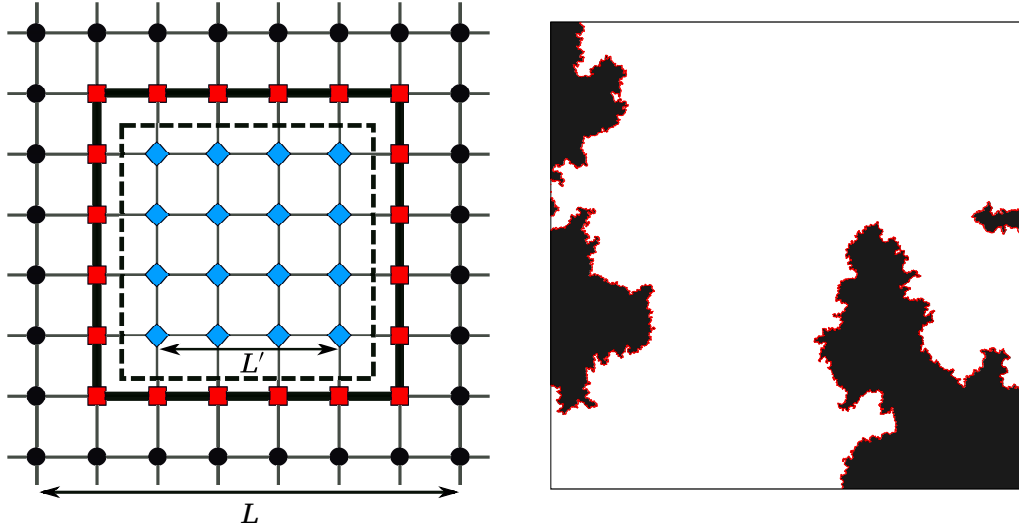
Figure 2. Left: Setup used for the windowing technique to compute ground states for samples with fully periodic boundary conditions.. Right: The domain wall separating regions of equal and opposite ground-state configurations for a specific disorder sample considered with periodic and with antiperiodic boundaries.

independent runs of this type is [30, 29]

$$P_s(\{J_{ij}\}) = 1 - [1 - P_n(\{J_{ij}\})]^m. \tag{2}$$

Studying these dependencies systematically [29], we find that 8 independent runs with 10 minimization steps with randomly chosen window locations yield a total success probability of $P_s = 0.999$, independent of system size. If desired, this can be further increased performing additional independent runs. Through the independence of the success probability from the system size, this setup results in an algorithm that produces exact ground states with a prescribed success probability with a constant-factor overhead over the underlying MWPM algorithm which is known to be polynomial [20].

3. Defect energies

The behavior of defects in the ordered state is among the fundamental differences in the various models that have been proposed to describe the spin-glass state [2]. Banavar and Cieplak [31] were the first to propose a study of defect energies as a means to understand the nature of the spin-glass phase. In particular, it was argued to consider the energy difference between the ground states of systems with periodic and antiperiodic boundary conditions, for which a scaling according to

$$E_{\text{def}} \propto L^\theta, \tag{3}$$

was proposed [32]. Generalizing Peierls' argument for the stability of the ordered phase, it was argued that a spin-stiffness exponent $\theta < 0$ corresponds to a situation were droplet excitations are unstable against thermal fluctuations and hence the spin-glass phase is not stable at non-zero temperatures. For the opposite case $\theta > 0$, on the other hand, spin glass should be able to exist for $0 < T \leq T_g$.

We studied defect energies by changing the boundary conditions for individual samples from periodic to antiperiodic and considering the resulting difference in ground-state energies, i.e,

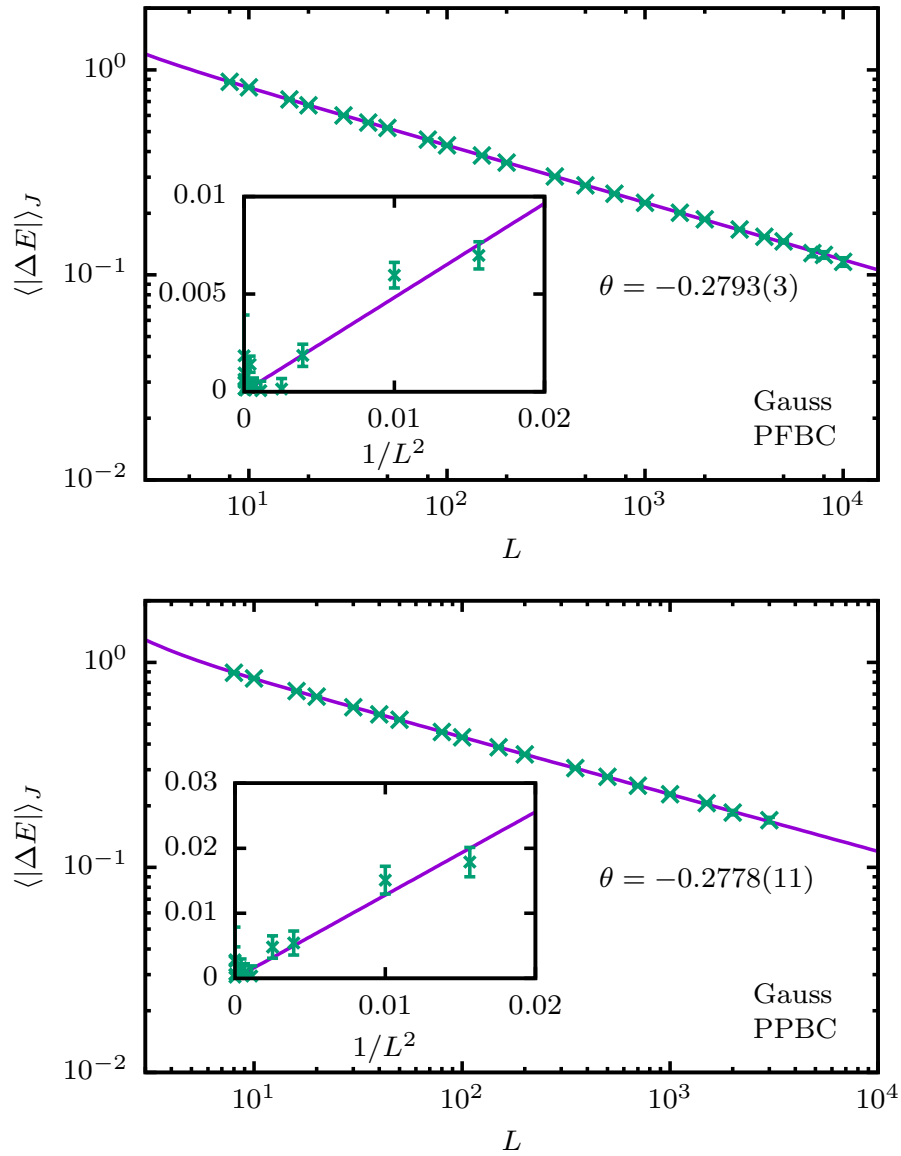$$E_{\text{def}} = |E_{\text{P}} - E_{\text{AP}}|.$$

Figure 3. Average defect energies $\langle|\Delta E|\rangle_J$ for the square-lattice Ising spin glass with Gaussian interactions as calculated from the difference in ground-state energies between periodic and antiperiodic boundary conditions in the x direction, and with free boundaries in y direction (PFBC, top) and periodic boundaries in y direction (PPBC, bottom), respectively. The lines show fits of the functional form (4) to the data. The insets show the correction $\langle|\Delta E|\rangle_J(L)-A_\theta L^\theta$ plotted against $1/L^2$ illustrating that this single term describes the corrections very well.

In the other lattice direction we either chose free boundaries, leading to periodic-free boundary conditions (PFBC), or also periodic boundaries (PPBC), where in the latter case we made use of the newly developed algorithm for fully periodic lattices based on the windowing technique [29]. The resulting defect energies, averaged over up to 10^6 disorder samples for the smallest system sizes, and for systems of up to $10\,000 \times 10\,000$ and 3000×3000 spins for the cases of PFBC and PPBC, respectively, are shown in Fig. 3. It is clear that the data follow the power-law implied

by Eq. (3) rather clearly, but there are some small corrections to scaling that, as the insets of Fig. 3 reveal, are well described by a single $1/L^2$ correction. We hence performed fits of the functional form

$$\langle |\Delta E| \rangle_J(L) = A_\theta L^\theta + C_\theta/L^2 \tag{4}$$

to the data, finding very good fit qualities [33] of $Q = 0.16$ for PFBC and $Q = 0.92$ for PPBC, including systems with $L \geq 8$ for PFBC and $L \geq 10$ for PPBC. The resulting estimates of the stiffness exponent, $\theta = -0.2793(3)$ (PFBC) and $\theta = -0.2778(11)$ (PPBC), respectively, are in excellent agreement with each other and of significantly higher accuracy than previous estimates of the stiffness exponent of this model, see, e.g., Ref. [34].

4. Domain wall fractal dimension

The change of boundary conditions considered above induces a change in the ground-state spin configuration in some part of each sample. If one draws lattice sites with unchanged ground-state spin configurations in black and lattice sites with overturned spin configurations in white, the typical configurations of such overlaps look like what is depicted in the right panel of Fig. 3. The boundary between the two domains forms a fractal curve with a Hausdorff dimension $1 < d_\mathrm{f} < 2$ [35]. The data for the domain-wall lengths for PFBC and PPBC boundary conditions are shown in the upper and lower panel of Fig. 4, respectively. We find the lengths to follow a power law up to very high accuracy, and as the insets of Fig. 4 show, it is hardly possible to resolve any corrections to this leading dependence. We hence performed fits to the pure power-law behavior,

$$\langle \ell \rangle_J = A_\ell L^{d_\mathrm{f}}, \tag{5}$$

to the data for $L \geq L_\mathrm{min} = 40$. The fractal dimensions extracted from these fits are $d_\mathrm{f} = 1.273\,19(9)$ for PFBC and $d_\mathrm{f} = 1.2732(5)$ for PPBC, respectively. As is seen from the insets of Fig. 4, scaling corrections for the PPFC geometry are even smaller than those for PFBC, an observation that is in line with the general expectation of reduced scaling corrections for fully periodic systems. The two values extracted from PFBC and PPBC are well compatible with each and, again, significantly more accurate than previously published estimates, see, e.g., Ref. [36].

5. Domain walls as stochastic Loewner evolution

The fact that the continuum limit of spin systems at a critical point is scale invariant is at the heart of our understanding of critical phenomena and forms the basis for the renormalization group [37]. The fact that there is also rotational symmetry and, more generally, *local* scale invariance leads to the notion of conformal symmetry, that is a powerful mathematical instrument for the description of critical systems [38]. While there are results in three dimensions [39–42], in two dimensions, specifically, the conformal group is infinite-dimensional since any holomorphic map is conformal, and hence conformal invariance alone is able to fully determine the form of certain correlation functions and in many cases gives exact results for all critical exponents. While this has enabled a comprehensive characterization of the operator content of all standard universality classes in two dimensions, this approach runs into difficulties for systems with quenched disorder that, prior to performing the disorder average, do not have conformal or, in fact, even translational invariance. Conformal field theories describing spin glasses, random-field problems or even systems with weak disorder such as disordered ferromagnets in two dimensions have hence not been found [38].

A complementary approach to critical lattice systems focused on geometrical aspects is given by stochastic Loewner evolution (SLE). It allows to connect the geometry of lattice paths such as domain boundaries of critical systems to the statistics of one-dimensional Brownian motion [43], thus resulting in a wide range of exact results for geometrical properties of such paths in
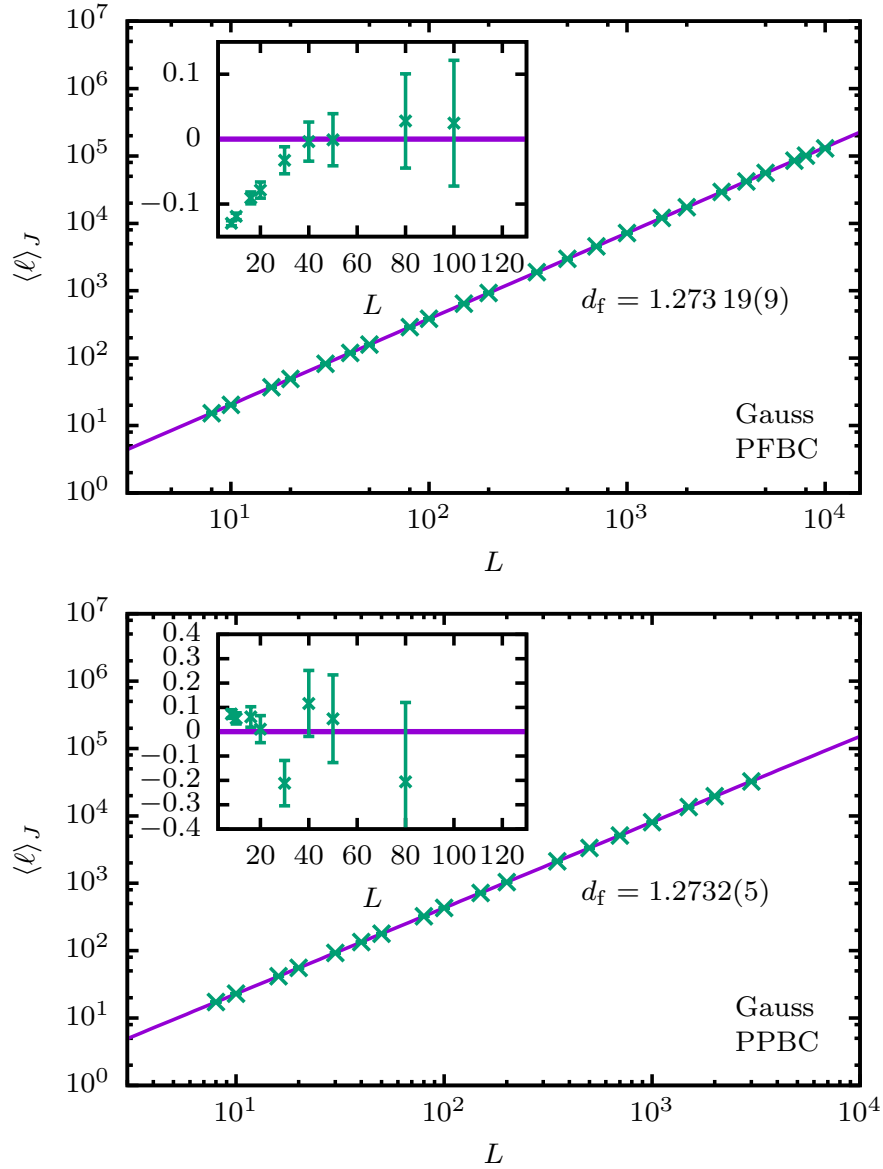
Figure 4. Average length of the domain wall in the 2D Edwards-Anderson Ising spin glass with Gaussian couplings induced by a change of boundary conditions from periodic to antiperiodic. The lines show fits of the functional form (5) to the data. The insets indicate the deviations $L^{d_{\mathrm{f}}} - \langle \ell \rangle_J$.

the scaling limit. Domain boundaries in percolation and the Ising model, for example, have been shown rigorously to be described by SLE [44, 45]. Domain walls in disordered systems have also been investigated regarding compatibility or not with SLE [46–49], and in some cases suprising consistencies have been found, nourishing hopes of the construction of conformal field theories for such systems.

One well-known prediction of SLE regards the probability of an SLE trace to pass to the left of a given point in the domain under consideration [43]. While SLE is normally defined in the upper half plane, the fact that all holomorphic maps in two dimensions are conformal means that
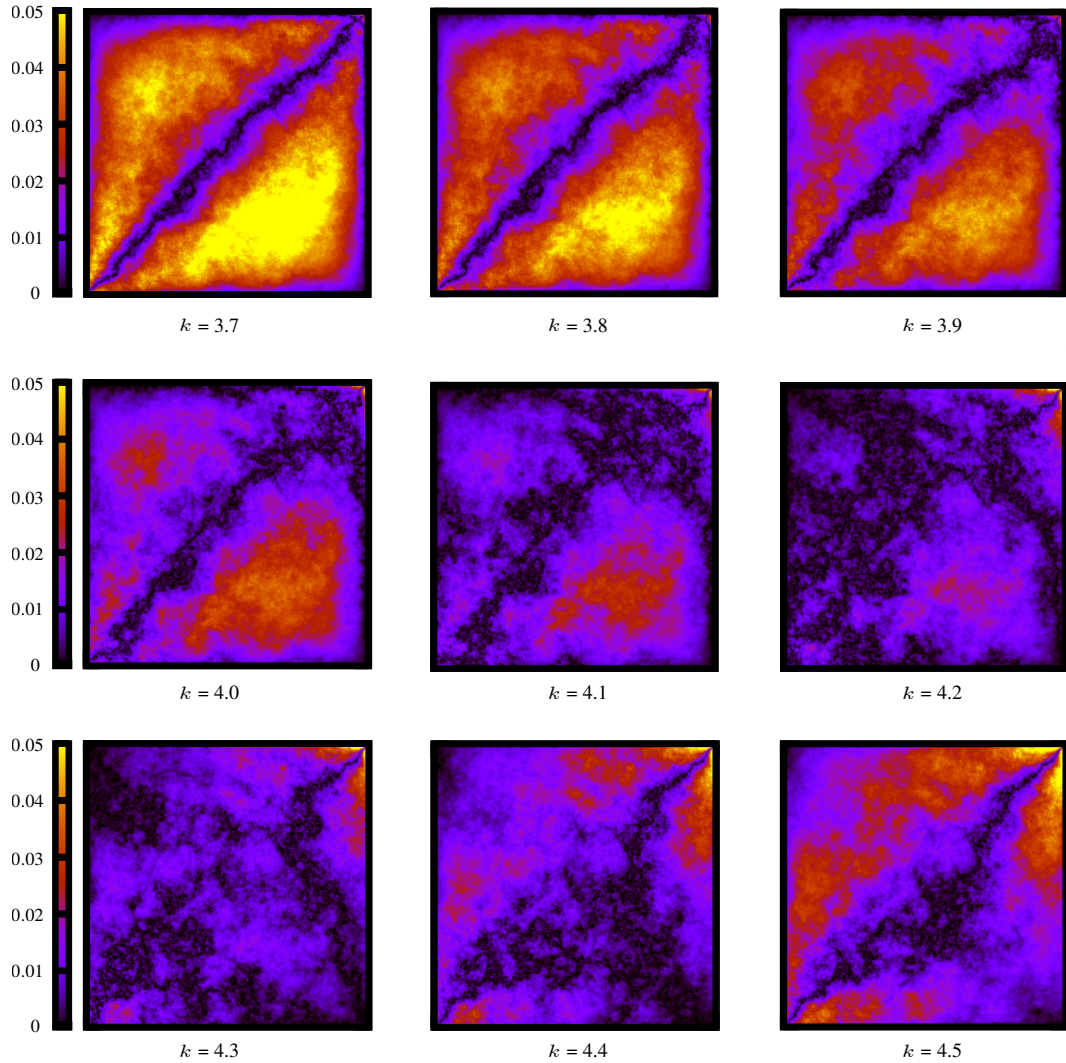
Figure 5. Average deviations of the observed left-passage probabilities of domain walls in the Gaussian 2D Ising spin glass from the exact results for SLE traces derived from the expression (6) for the upper half plane for different values of κ. The data are for systems with 100×100 spins.

one can also consider the problem in any other geometry that can be conformally connected to the upper half plane, including for instance a system of square shape as it is normally considered in lattice systems. The left-passage probability of an SLE trace of parameter κ in the upper half plane is given by [50]

$$P_{\mathrm{LP}}^{\kappa}(x,y) = \frac{1}{2} + \frac{\Gamma(4/\kappa)}{\sqrt{\pi}\,\Gamma\left(\frac{8-\kappa}{2\kappa}\right)} \frac{x}{y}{}_2F_1\left(\frac{1}{2}, \frac{4}{\kappa}; \frac{3}{2}; -\left(\frac{x}{y}\right)^2\right),$$

(6)

and the corresponding expression for the domain at hand can be calculated from the conformal transformation connecting the two geometries.

While a general consistency of domain walls in the two-dimensional spin glass with SLE had already been observed in Refs. [46, 47], here we study the dependence of these observations on

the boundary conditions used to force a domain wall into the system. Under the conformal map connecting the upper half plane and the square geometry considered for the spin model, the origin of the upper half plane is mapped onto the lower left corner of the square, while the infinite point of the plane is mapped onto the top right corner. The domain wall should hence be pinned to run exactly across diagonally opposing corners. This can be achieved by introducing strong bounds that make it impossible for the domain wall to break any couplings along the circumference of the square apart from the bonds at the two corner points of the system. In Fig. 5 we show the deviations of measured left-passage probabilities from the ones calculated from Eq. (6) under the assumption of different values of the diffusion constant κ. It is seen that a relative minimum of these deviations is attained for $\kappa \approx 4.3$, but it turns out that there are remaining systematic deviations even at that minimum. Additionally, κ is connected to the fractal dimension of domain walls as $d_{\mathrm{f}} = 1 + \kappa/8$ [43] and $\kappa \approx 4.3$ hence implies $d_{\mathrm{f}} \approx 1.54$, which is not compatible with the accurate estimates of d_{f} reported above. It hence appears clear that domain walls in the 2D Ising spin glass with fixed boundaries are not compatible with SLE, and only for effectively free boundaries as the ones studied in Refs. [46, 47] compatibility with SLE is possible. Sensitivity to boundary conditions is not unknown in the study of SLE as for example the loop-erased random-walk is compatible with SLE for certain boundary conditions but not for others [51]. Still, the observations reported here require additional work to come to a definite understanding of the relation of domain-wall boundaries in the 2D Ising spin glass to SLE traces.

6. Conclusions

Using a rarely used mapping of the Ising spin-glass ground-state problem on planar lattices to a minimum-weight perfect matching problem on a sparse graph we study ground-states of the system for a wide range of system sizes ranging up to $10\,000 \times 10\,000$ spins. With the help of a windowing algorithm, it is also possible to determine ground-states of systems with non-planar, fully periodic samples with just a constant-factor overhead over the polynomial run-time of the matching algorithm. Employing these techniques, we determine defect energies for the system and estimate a stiffness exponent $\theta = -0.2793(3)$ and a domain-wall fractal dimension $d_{\mathrm{f}} = 1.273\,19(9)$. Testing domain walls in the system for their compatibility with SLE traces, it is found that the result strongly depends on the chosen boundary conditions, and while SLE was found to apply for effectively free boundaries, it does not hold for fixed boundaries. Compatibility with SLE is hence a less universal property than, for example, a critical exponent.

Acknowledgments

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