Computational Physics with GPUs Lecture 2: A first course in CUDA

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Slides and exercises

Check out the lecture notes and example code at

http://users.complexity-coventry.org/~weigel/GPU/

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Example:

<pre>int *ip;</pre>	// declare variable // declare pointer // another pointer				
•	<pre>// store address of var in pointer // error! cp must be the address of an int</pre>				
<pre>printf("Address of var variable: %p\n", &var); printf("Address stored in ip variable: %p\n", ip); printf("Value of *ip variable: %d\n", *ip);</pre>					

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Some valid pointers:

int	*ip;	11	pointer	to	an integer
float	*fp;	11	pointer	to	a float
float	**fpp;	11	pointer	to	a pointer
void	*vp;	11	pointer	to	void

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Arrays in C are stored in sequential order, so pointers can be used to inspect and manipulate arrays:

```
int var[] = {1, 2, 3};
int *p;
p = &var[0]; // point to the first element
p = var; // an equivalent expression
++p; // p now points to the second element
if (p > var)
    printf("p points to an element further to the right than var");
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```

Strings in C are stored as arrays of characters:

```
char str[10] = "hello";
str[0]; // returns char 'h'
str; // address of the first character of the string
```

```
Sometimes it is even useful to have a pointer to a pointer!
int a = 2;
int *ap = &a;
int **app = ≈
printf("%d = %d = %d\n", a, *ap, **app);
```

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```

This is particularly useful for two-dimensional arrays: int **a;

```
a = (int **) malloc(10*sizeof(*int));
for(int i = 0; i < 10; ++i)
    a[i] = (int*) malloc(20*sizeof(int));
a[8][15] = 12;
```

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a = (int**)malloc(10*sizeof(*int));
for(int i = 0; i < 10; ++i)
    a[i] = (int*)malloc(20*sizeof(int));
a[8][15] = 12;</pre>
```

```
Pointers are needed in C to alter objects in functions:
void f1(int a) {
    a = 10;
  }
void f2(int *a) {
    *a = 10;
  }
int b = 20;
f1(b); printf("%d\n", b);
f2(&b); printf("%d\n", b);
```

Outline









GPU workflow

GPU workflow

The basic workflow for a GPU code proceeds along the following lines:

• Initialize input data (matrices, particle configurations, ...) on CPU.

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More elaborate variations:

• Use of unified virtual addressing.

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- Part or full off-loading of calculations to GPU.
- Interleaving computation and communication.

GPU workflow

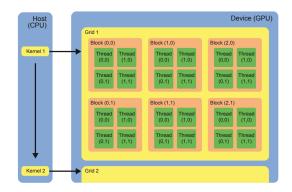
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- Execute kernel(s) on GPU, processing the data.
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- Use of unified virtual addressing.
- Part or full off-loading of calculations to GPU.
- Interleaving computation and communication.
- Integration with MPI.

Kernel execution

- Kernel GPU program that runs on a grid of threads
- Thread scalar execution unit
 - Warp block of 32 threads executed in lockstep
 - Block a set of warps executed on the same SM
 - Grid a set of blocks usually executed on different SMs



Execution configuration

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Modified C function call syntax:

kernel<<<dim3 dG, dim3 dB>>>(...);

Execution configuration:

- dg: dimension and size of grid in blocks
 - $\bullet~$ two-dimensional, ${\tt dG.x}$ and ${\tt dG.y}$
 - total number of blocks launched is $\tt dG.x \times \tt dG.y$
- dB: dimension and size of each block
 - two- or three-dimensional, dB.x, dB.y, and dB.z
 - total number of threads per block is dB.x \times dB.y \times dB.z
 - if not specified, dB.z = 1 is assumed

Built-in variables

All <u>__global__</u> and <u>__device__</u> functions have the following automatic variables:

- dim3 gridDim; dimension of the grid in blocks
- dim3 blockDim; dimension of the block in threads
- dim3 blockIdx; block index within grid
- dim3 threadIdx; thread index within block

The indices can be used to construct a global thread index, for instance for a block size of 5 threads,

```
thread_index = blockIdx.x*blockDim.
    x + threadIdx.x;
```

Possible grid dimensions are specified in the programming guide,

https://docs.nvidia.com/cuda/cuda-c-programming-guide/#compute-capabilities

Index 1

If we need to use each thread to calculate one output element of a vector addition, what would be the expression for mapping the thread/block indices to the data index?

- (a) i=threadIdx.x+threadIdx.y
- (b) i=blockIdx.x+threadIdx.x
- (c) i=blockIdx.x*blockDim.x+threadIdx.x
- (d) i=blockIdx.x*threadIdx.x

Quiz

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Index 2

We want to use each thread to calculate two adjacent elements of a vector addition. What mapping is correct if i is the index of the first element?

- (a) i=blockIdx.x*blockDim.x+threadIdx.x+2
- (b) i=blockIdx.x*threadIdx.x*2
- (c) i=(blockIdx.x*blockDim.x+threadIdx.x)*2
- (d) i=blockIdx.x*blockDim.x*2+threadIdx.x

Quiz

Possible grid dimensions are specified in the programming guide,

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Threads

For a vector addition, assume that the vector length is 2000, each thread calculates one output element, and the thread block size is 512 threads. How many threads will be in the grid?

- (a) 2000
- (b) 2024
- (c) 2048
- (d) 2096

Outline









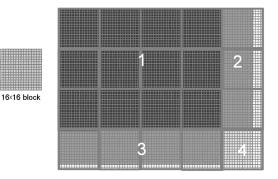
Thread mapping

The organization of threads in a (1D or 2D) grid of (1D, 2D or 3D) blocks is usually chosen to match the dimensionality and structure of the input data.

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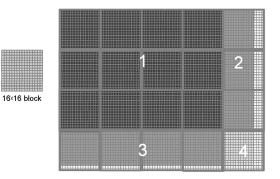
For example, a 2D grid of 2D blocks would probably be used to work on the pixels of an image.



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Note that (dynamic) arrays in C need to be *linearized*: $M_{ij} = M[i * n + j]$ (row-major).

Example: matrix multiplication

Consider multiplication of two square, <code>Width</code> \times <code>Width</code> matrices <code>d_M</code> and <code>d_N</code>, <code>d_P = d_M \cdot d_N</code>.

The grid

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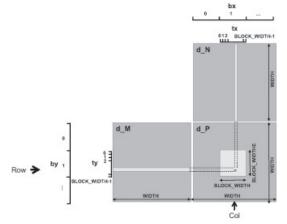
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Matrix multiplication: a simple kernel

Matrix multiplication kernel

```
__global__ void MatrixMult(float* d_M, float* d_N, float* d_P, int Width) {
    // Calculate the row index of the d_P element and d_M
    int Row = blockIdx.y*blockDim.y+threadIdx.y;
    // Calculate the column index of d_P and d_N
    int Col = blockIdx.x*blockDim.x+threadIdx.x;
    if ((Row < Width) && (Col < Width)) {
      float Pvalue = 0;
      // each thread computes one element of the block sub-matrix
      for (int k = 0; k < Width; ++k) {
           Pvalue += d_M[Row*Width+k]*d_N[k*Width+Col];
      }
      d_P[Row*Width+Col] = Pvalue;
    }
}</pre>
```

Matrix multiplication: a simple kernel (cont'd)

In the host code, the split of the matrix into blocks needs to be organized, probably using an adjustable block size

(Part of) the host code

```
#define BLOCK_WIDTH 16
```

```
int NumBlocks = Width/BLOCK_WIDTH;
if(Width % BLOCK_WIDTH) NumBlocks++;
dim3 dimGrid(NumBlocks, NumBlocks);
dim3 dimBlock(BLOCK_WIDTH, BLOCK_WIDTH);
```

MatrixMult << <dimGrid, dimBlock >>>(Md, Nd, Pd, Width);

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dim3 dimBlock(BLOCK_WIDTH, BLOCK_WIDTH);
MatrixMult<<<dimGrid, dimBlock>>>(Md, Nd, Pd, Width);
```

A few notes are in order:

- we need to ensure that we have enough threads to cover all matrix elements
- the excess threads are then "masked away" in the kernel code
- one might want to optimize over difference choices of BLOCK_WIDTH for best performance

Thread scheduling and synchronization

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These principles lead to good results for devices with very different scopes of available resources, known as transparent scalability.

To understand the effect of scheduling, one needs to take into account the resource limits of the multiprocessors, which can be checked at

https://docs.nvidia.com/cuda/cuda-c-programming-guide/#compute-capabilities

(And they can be queried using cudaGetDeviceProperties().)

Resident threads

If a CUDA device's SM can take up to 1536 threads and up to 4 thread blocks, which of the following block configurations would result in the most number of threads in the SM?

- (a) 128 threads per block
- (b) 256 threads per block
- (c) 512 threads per block
- (d) 1024 threads per block

Quiz

Threads in the grid

For a vector addition, assume that the vector length is 2,000, each thread calculates one output element, and the thread block size is 512 threads. How many threads will be in the grid?

- (a) 2,000
- (b) 2,024
- (c) 2,048
- (d) 2,096

Quiz

Scheduling on CUDA devices works in warps of 32 threads that operate in lockstep, always operating exactly the same code. If a conditional evaluates differently for some threads in a warp, the code needs to run multiple times, once for each outcome. This is called thread divergence.

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Warps For the previous question, how many warps do you expect to have divergence due to the

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(a) 1
(b) 2
(c) 3
(d) 6

Outline







4 A worked example

Memory accesses are most often the performance limiting factor in GPU applications.

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Memory

In the first matrix multiplication kernel, the main loop is

```
for (int k = 0; k < Width; ++k) {
    Pvalue += d_M[Row*Width+k]*d_N[k*Width+Col];</pre>
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which loads two values, stores one value and performs a multiplication and an addition. Hence the *compute to global memory access ratio* is 1.

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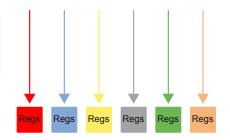
Imagine we are using a card which has a memory bandwidth of about 200 GB/s and a peak single precision FP performance of 1500 GFlop/s.

With 4 bytes per float, the maximum performance of this kernel is 200/4 = 50 GFlop/s, which is just 3% of the peak performance! To arrive at the peak performance, we would need 30 FP operations per global memory access.

Memory hierarchy

Per thread

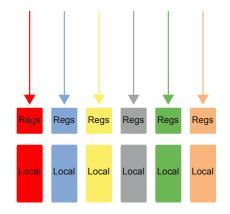
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Memory hierarchy

Per thread

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Memory hierarchy

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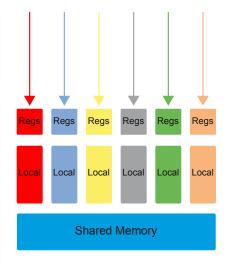
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Thread blocks: shared memory

- allocated by thread block, same lifetime as block
- allocate as

__shared__ int s_array[DIM];

- low latency (of the order of 10 cycles), bandwidth up to 1 TB/s
- use for data sharing and user-managed cache



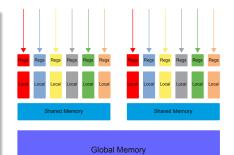
Memory hierarchy

Per device: global memory

- accessible to all threads on device
- lifetime is user-defined

```
cuda_malloc(void **pointer,
    size_t nbytes);
cuda_free(void* pointer);
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- latency several hundred clock cycles
- bandwidth ≈ 160 GB/s on Fermi (access pattern needs to conform to coalescence rules for good performance)



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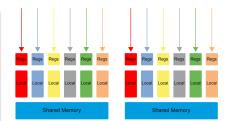
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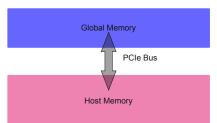
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Per host: device memory

- no direct access from CUDA threads
- copy data to/from device with

```
cudaMemcpy(void* dest, void*
    src, size_t nbytes,
    cudaMemcpyHostToDevice);
```



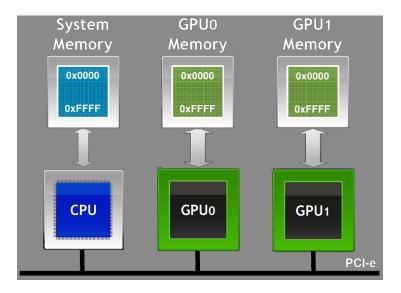


Memory hierarchy (summary)

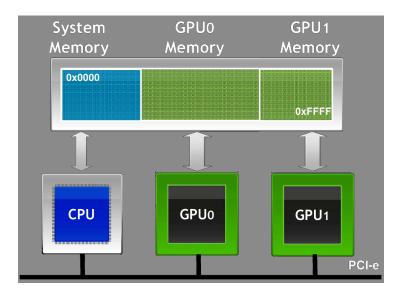
More generally, the different types of memory have the following characteristics:

Memory	Location	Cached	Access	Scope	Lifetime
Register	On-chip	N/A	R/W	One thread	Thread
Local	Off-chip	No	R/W	One thread	Thread
Shared	On-chip	N/A	R/W	All threads in a block	Block
Global	Off-chip	(Yes)	R/W	All threads $+$ host	Application
Constant	Off-chip	Yes	R	All threads + host	Application
Texture	Off-chip	Yes	R	All threads $+$ host	Application

Unified virtual addressing



Unified virtual addressing



CUDA variables

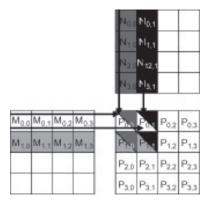
Variable declaration	Memory	Scope	Lifetime	Penalty/Latency
int var;	register	thread	thread	1X
<pre>int array_var[10];</pre>	local	thread	thread	100X (pre-Fermi)
shared int shared_var;	shared	block	block	10X
device int global_var;	global	grid	application	100X
constant int constant_var;	constant	grid	application	1X

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device int global_var;	global	grid	application	100X
constant int constant_var;	constant	grid	application	1X

- automatic scalar variables reside in registers, compiler will spill into local memory in shortage of registers
- automatic array variables (in the absence of qualifiers) reside in thread-local memory
- the type of memory used will be crucial for the performance of the application

In the way we have set up the multiplication kernel,

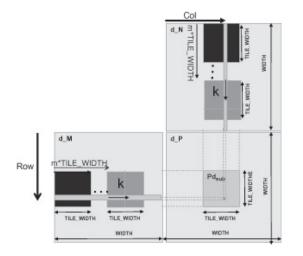


Memory

each value will be loaded Width times from global memory!

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The tile width is mostly limited by the shared memory size: we need $4 \times 2 \times \text{TILE}_WIDTH$ bytes, e.g., a width of 64 results in 32K of shared memory used.

Maxtrix multiplication (again)

Matrix multiplication kernel

```
__global__ void MatrixMult(float* d_M, float* d_N, float* d_P, int Width) {
  __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];
  shared float Nds[TILE WIDTH][TILE WIDTH];
  int bx = blockIdx.x, by = blockIdx.y;
  int tx = threadIdx.x, ty = threadIdx.y;
 // Identify the row and column of the d P element
  int Row = bv*TILE WIDTH+tv. Col = bx*TILE WIDTH+tx:
  float Pvalue = 0;
  // Loop over d M and d N tiles
  for (int m = 0: m < Width/TILE WIDTH: ++m) {</pre>
    // Collaborative loading of tile
    Mds[tv][tx] = d M[Row*Width+m*TILE WIDTH+tx]:
    Nds[ty][tx] = d_N[(m*TILE_WIDTH+ty)*Width+Col];
    syncthreads();
    for (int k = 0; k < TILE WIDTH; ++k)
      Pvalue += Mds[ty][k] * Nds[k][tx];
     svncthreads():
  }
  d P[Row*Width+Col] = Pvalue:
}
```

Memory

Outline









Definition

Let f(z) = p(z)/q(z) be a complex function, where p(z) and q(z) are complex polynomials.

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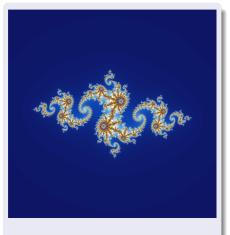
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$$f(x) = z^2 + c_i$$

where c is a complex constant.



(Color codes are for different rates of divergence.)

Julia: CPU code 1

Driver

```
int main( void ) {
    CPUBitmap bitmap( DIM, DIM );
    unsigned char *ptr = bitmap.get_ptr();
    kernel( ptr );
    bitmap.display_and_exit();
}
```

- CPUbitmap externally defined
- display_and_exit
 () externally
 defined

Julia: CPU code 1

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    CPUBitmap bitmap( DIM, DIM );
    unsigned char *ptr = bitmap.get_ptr();
    kernel( ptr );
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}
```

Kernel

```
void kernel( unsigned char *ptr ){
  for (int y=0; y<DIM; y++) {
    for (int x=0; x<DIM; x++) {
        int offset = x + y * DIM;
        int juliaValue = julia( x, y );
        ptr[offset*4 + 0] = 255 * juliaValue;
        ptr[offset*4 + 1] = 0;
        ptr[offset*4 + 2] = 0;
        ptr[offset*4 + 3] = 255;
      }
  }
}</pre>
```

- CPUbitmap externally defined
- display_and_exit
 () externally
 defined

- julia() will return 1 if point in set
- choose red and black colors, respectively

Julia: CPU code 2

Julia set function

```
int julia( int x, int y ) {
    const float scale = 1.5;
    float jx = scale * (float)(DIM/2 - x)/(DIM/2);
    float jy = scale * (float)(DIM/2 - y)/(DIM/2);
    cuComplex c(-0.8, 0.156);
    cuComplex a(jx, jy);
    int i = 0:
    for (i=0: i<200: i++) {</pre>
        a = a * a + c;
        if (a.magnitude2() > 1000)
           return 0;
    }
    return 1;
}
```

Julia: CPU code 2

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    const float scale = 1.5:
    float jx = scale * (float)(DIM/2 - x)/(DIM/2);
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            return 0;
    ł
    return 1;
}
```

- shift center of image to (0,0)
- re-scale to unit square in complex plane
- use scale factor to zoom

• arbitrary constant
$$c$$
 in $z_{n+1} = z_n^2 + c$

Julia: CPU code 3

Data structure

```
struct cuComplex {
    float r;
    float i;
    cuComplex(float a, float b) : r(a), i(b) {}
    float magnitude2( void ) { return r * r + i * i; }
    cuComplex operator*(const cuComplex& a) {
        return cuComplex(r*a.r - i*a.i, i*a.r + r*a.i);
    }
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    }
};
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Julia: CPU code 3

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        return cuComplex(r+a.r, i+a.i);
    }
};
```

- use cuComplex structure to represent complex numbers
- operator overloading for natural semantics

Julia: GPU code 1

Driver

}

```
int main( void ) {
    DataBlock data:
    CPUBitmap bitmap( DIM, DIM, &data );
    unsigned char *dev_bitmap;
    HANDLE ERROR( cudaMalloc( (void**)&dev bitmap, bitmap.image size() ) );
    data.dev_bitmap = dev_bitmap;
    dim3
            grid(DIM,DIM);
    kernel <<< grid .1>>>( dev bitmap ):
    HANDLE_ERROR( cudaMemcpy( bitmap.get_ptr(), dev_bitmap,
                              bitmap.image size().
                              cudaMemcpyDeviceToHost ) );
    HANDLE_ERROR( cudaFree( dev_bitmap ) );
    bitmap.display_and_exit();
```

Julia: GPU code 1

Driver

}

```
int main( void ) {
    DataBlock data:
    CPUBitmap bitmap( DIM, DIM, &data );
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    bitmap.display_and_exit();
```

- use cudaMalloc to allocate memory on device
- assign one thread per pixel, one thread per block, resulting in a DIM \times DIM grid
- third dimension in dim3 defaults to one \Rightarrow 2D grid

Julia: GPU code 2

Kern<u>el</u>

```
__global__ void kernel( unsigned char *ptr ) {
    // map from blockIdx to pixel position
    int x = blockIdx.x;
    int y = blockIdx.y;
    int offset = x + y * gridDim.x;
    // now calculate the value at that position
    int juliaValue = julia( x, y );
    ptr[offset*4 + 0] = 255 * juliaValue;
    ptr[offset*4 + 1] = 0;
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}
```

Julia: GPU code 2

Kernel

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}
```

- __global__ qualifier for device function to be called from host
- each thread gets threadIdx.x, threadIdx.y, blockIdx.x, and blockIdx.y
- translate to offset in image array
- note that for loops have gone away
- four chars to represent R, G, B and alpha channels

Julia: GPU code 3

Julia function

```
__device__ int julia( int x, int y ) {
    const float scale = 1.5;
    float jx = scale * (float)(DIM/2 - x)/(DIM/2);
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    cuComplex c(-0.8, 0.156);
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    int i = 0;
    for (i=0; i<200; i++) {
        a = a * a + c;
        if (a.magnitude2() > 1000)
            return 0;
    }
    return 1;
}
```

Julia: GPU code 3

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}
```

- __device__ qualifier for device function to be called from device code
- identical to CPU code apart from function qualifier

Julia: GPU code 4

Data structure

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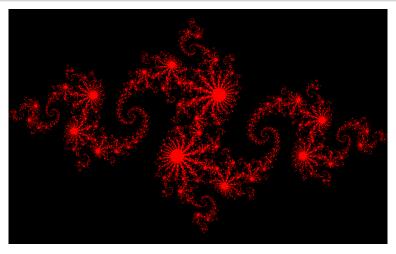
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    };
};
```

- almost identical to CPU version
- only difference are __device__ function qualifiers

Julia: result



Play around with code, cf.

J. Sanders, E. Kandrot: "*CUDA by example — An Introduction to General-Purpose GPU Programming*", (Addison Wesley, Upper Saddle River, 2011).

M. Weigel (Coventry/Mainz)

Julia: GPU code — improvements

• On Fermi, maximum number of resident blocks per SM is 8 (16 for Kepler), hence 24 out of 32 cores (176 out of 192 for Kepler) are idle

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- $\bullet\,$ To improve, one could introduce tiles of, say, 16×16 pixels. How would the program need to be modified?
- could have used virtual unified addressing to eliminate the CPU copy of the image
- could integrate with OpenGL for interactive rendering

• . . .

Summary and outlook

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Reading

If using the book by Kirk and Hwu, you could work through all material until Chapter 5. More about the Julia set problem on GPU can be found in Sanders and Kandroot: *CUDA by example*.